Announcements

1) Practice problems for Quiz #2 up on CTools under "Assignments"

2) Quiz Thursday, covers 3.1 and 3.3.

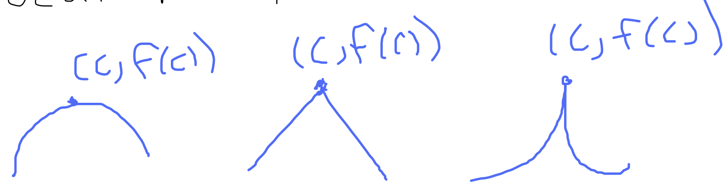
3) Intermediate Value Theorem notes (10/3/11) now on CTools.

Definition (local max/min)

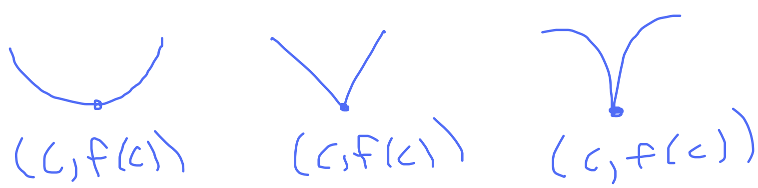
f has a local maximum at $x=c$ if there is an open interval I containing c with $f(x) \le f(c)$ for all x in I. f has a local minimum at $x=d$ if there is an open interval J containing d with $f(x) \ge f(d)$ for all x in J.

Pictures

Local maxima



Local minima



Fermat's Theorem: If f has a local maximum or local minimum at $x = c$, then if $f'(c)$ exists,

$$f'(c) = 0.$$

(tangent line is horizontal)

Warning: Just because

$f'(c) = 0$ does not mean

f has either a local max

or a local min at $x = c$.

Counterexample: $f(x) = x^3$.

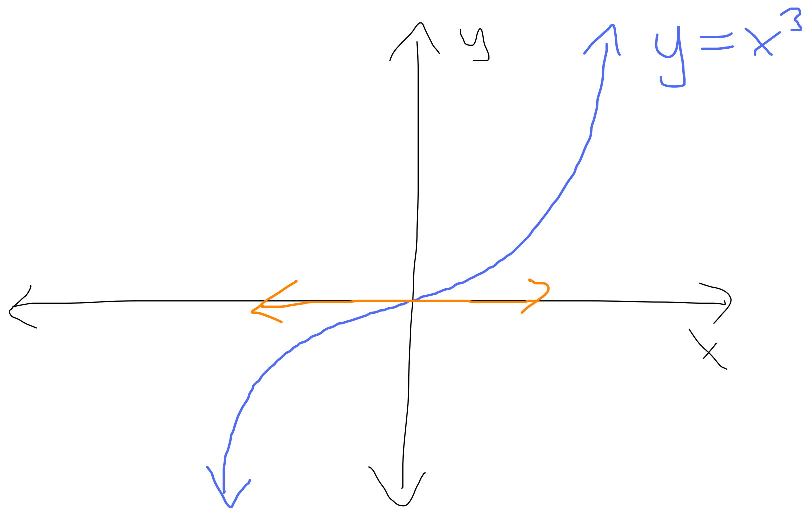
$f'(x) = 3x^2$, $f'(0) = 0$.

But f has neither a

local max nor a local min

at $x = 0$.

Picture



Digression: x, y, and z are natural numbers

$x^2+y^2=z^2$ has solutions (Pythagorean Theorem)

What about $x^3+y^3=z^3$

or $x^6+y^6=z^6$

or more generally,

$x^n+y^n=z^n$ for n a natural number $(n \geq 3)$

Fermat said that you never have $x^n + y^n = z^n$ for natural numbers x, y, z if $n > 2$.

It took 400 years for someone to prove this (Andrew Wiles)

$\sim$ 300 pages of math

Facts About Continuous Functions

Let f be continuous on a closed interval $[a,b]$.

Then there are numbers c and d in $[a,b]$ with

$$f(d) \leq f(x) \leq f(c)$$

for all x in $[a,b]$.

Idea: Continuous function on closed intervals have absolute maxima and minima on those intervals.

Warning: Doesn't work for open intervals: just take $f(x) = x$ on $(0,1)$.

Definition: A point $x = c$ in the domain of a function f is called a critical number if either $f'(c) = 0$ or $f'(c)$ does not exist.

Finding Absolute Max/Min on a Closed Interval

1) Take the derivative, set equal to zero, find all zeros of the derivative.

2) Find all points where the derivative does not exist.

3) Take all points found in steps 1) and 2), throw in the endpoints of the interval, plug all these points back into the original function. The biggest y-value is the max, the smallest is the min.

For quadratic

$$f(x) = 2x^3 - 3x^2 - 36x + 4,$$

Consider on the interval

$[-5, 1]$. Find

the maxima and minima

on $[-5, 1]$.

1) $f'(x) = 6(x-3)(x+2)$

$$0 = 6(x-3)(x+2)$$

$$x = \cancel{3}, -2$$

We don't include $x=3$

since 3 is not in $[-5,1]$.

We only have $x=-2$.

2) f' is defined everywhere,

so there are no points

where f' does not exist.

3) Points are $x=-2,-5,1$.

$f(-2)= 48$

$f(-5)=-141$

$f(1) = -33$

So max is 48 $(x=-2)$,

min is -141 $(x=-5)$.

Example 1: $|\sin(x)| = f(x)$

on $\left[-\frac{\pi}{3}, \frac{\pi}{6}\right]$.

1) $f'(x) = ?$

$$= \begin{cases} \cos(x), & x > 0 \\ -\cos(x), & x < 0 \\ \text{undefined}, & x = 0 \end{cases}$$

at $x = 0$, we have

to use the definition!

$$f'(0) = \lim_{h \to 0} \frac{f(h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{|\sin(h)|}{h}$$

If $h > 0$, $\sin(h) = |\sin(h)|$,

so $\displaystyle \lim_{h \to 0^+} \frac{|\sin(h)|}{h} = \lim_{h \to 0^+} \frac{\sin h}{h}$

$$= 1$$

$$\lim_{h \to 0^-} \frac{|\sin(h)|}{h} = \lim_{h \to 0^-} \frac{-\sin(h)}{h}$$

$$= -1$$

So $f'(0)$ does not exist.

Is it possible to have $f'(x)=0$ when x is in $\left[-\frac{\pi}{3}, \frac{\pi}{6}\right]$?

You'd need $\cos(x)=0$,

Never happens on our interval, so no points where $f'(x)=0$.

2) f' is undefined at

$x=0$ by previous

calculus.

3) Points are $x=0, -\frac{\pi}{3}, \frac{\pi}{6}$,

Plug into f.

$f(0) = |\sin(0)| = 0$

$f(-\frac{\pi}{3}) = |\sin(-\frac{\pi}{3})| = \frac{\sqrt{3}}{2}$

$f(\frac{\pi}{6}) = |\sin(\frac{\pi}{6})| = \frac{1}{2}$

Example 2: $-2x + \tan(x) = f(x)$

on $[0, \frac{\pi}{3}]$.

$$f'(x) = -2 + \sec^2(x)$$

1) $f'(x) = 0$

$$-2 + \sec^2(x) = 0$$

$$\sec^2(x) = 2$$

$$\sec(x) = \pm\sqrt{2}$$

same as

$$\cos(x) = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$$

only $x = \frac{\pi}{4}$ satisfies

$f'(x) = 0$ and is

in $[0, \frac{\pi}{3}]$.

so just $x = \frac{\pi}{4}$.

2) $f'(x) = \sec^2(x) - 2$

$$= \frac{1}{\cos^2(x)} - 2$$

ok on all of $[0, \frac{\pi}{3}]$

No points where f'

is undefined.

3) Points are $x = 0, \frac{\pi}{4}, \frac{\pi}{3}$

Plug in to f.

$(f(x) = -2x + \tan(x))$

$f(0) = 0$

$f\left(\frac{\pi}{4}\right) = -2\left(\frac{\pi}{4}\right) + \tan\left(\frac{\pi}{4}\right)$

$= -\frac{\pi}{2} + 1 < 0$

$f\left(\frac{\pi}{3}\right) = -\frac{2\pi}{3} + \tan\left(\frac{\pi}{3}\right)$

$= -\frac{2\pi}{3} + \sqrt{3} < 0$

Maximum is at

$x = 0$ $(f(0) = 0)$

Minimum at $x = \frac{\pi}{4}$

$(\approx -.57)$